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Introduction to neutron reflection

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Outline

Interference of waves
Refractive index
Critical angle, total reflection

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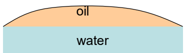
Reflection

Light



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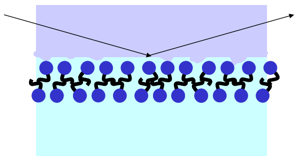
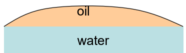
Reflection



Light



Why neutrons?

Contrast: light elements, isotopes
Penetrate
Magnetism

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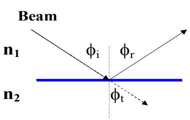
Reflection and Refraction: Snell's Law

Optical Notation



For specular reflection:

$\phi_i = \phi_r$

Transmitted beam is refracted:

$n_2 \sin \phi_t = n_1 \sin \phi_i$

n is refractive index

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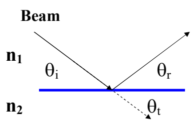
Reflection and Refraction: Snell's Law

Neutron Reflection Notation



$\theta = 90° - \varphi$

For specular reflection:

$\theta_i = \theta_r$

Transmitted beam is refracted:

$n_2 \cos \theta_t = n_1 \cos \theta_i$

n is refractive index

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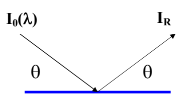
Reflection – measured quantities

Reflection



Reflected beam
deflected: 2 θ

Reflectivity
$R(Q) = I_R/I_0(\lambda)$

Momentum transfer
$Q = (4\pi/\lambda) \sin\theta$

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Demonstration Calculations

www.ncnr.nist.gov/instruments/magik/calculators/reflectivity-calculator.html

www.ncnr.nist.gov/instruments/magik/calculators/magnetic-reflectivity-calculator.html

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Critical Angle and Below (critical wavelength and above)

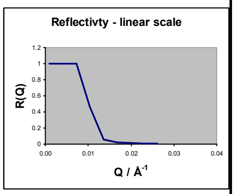
Density difference between two bulk phases determines the critical momentum transfer/angle, Q_c or θ_c

Any variation in intensity below critical angle is probably telling you about the experiment rather than the interface

$R(Q) = 1$ for $\theta < \theta_c$ is often used as a calibrant

$R(Q) \sim 1/Q^4$ for sharp interface

Total reflection below critical angle θ
$\cos\theta = n_2/n_1$



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Calculating Refractive Index

Neutrons

$$n = 1 - (\lambda^2 \Sigma_i b_i/V / 2\pi)$$

λ is the wavelength

$\Sigma_i b_i$ is the sum of scattering lengths in volume V

b is known for most stable nuclei

$\rho = \Sigma_i b_i/V$

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Scattering Lengths of Nuclei

Nucleus	Scattering Length / fm
^1H	-3.741
^2H (or D)	6.675
C	6.648
O	5.805
Si	4.151
Cl	9.579

Source: H. Rauch & W. Waschkowski

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Properties of Common Materials

Material	Scatt. Length Density / 10^{-6} Å^{-2}	Refractive index at 10 Å
H_2O	-0.56	1.000009
D_2O	6.35	0.999899
Si	2.07	0.999967
Air	0	1.000000
Polystyrene	1.4	0.999971

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Contrast in a Thin Film

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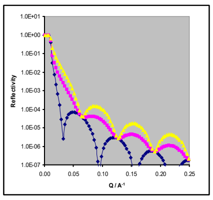
Calculation for Neutrons

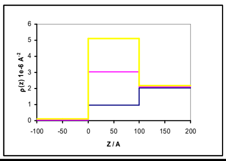
100 Å layer with ρ=1, 3 & 5 x 10^{-6} Å^{-2} on Si (ρ=2.07 x 10^{-6} Å^{-2})

Increasing contrast changes visibility of fringes

Phase change makes large difference

Fringes (Kiessig fringes) – spacing indicates film thickness for a single layer.





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Roughness

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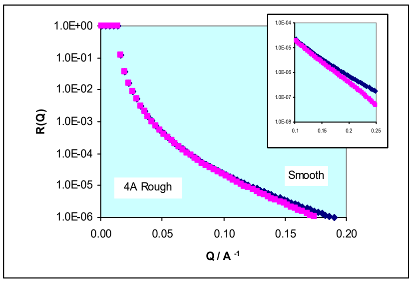
Reflectivity from rough surfaces is decreased.



L. Nevot, P. Crocé *J. Phys. Appl.* **15**, T61 (1980)

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Intensity of Reflected Signal

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Waves interfere constructively for

2 d sin θ = λ, 2λ, 3λ ... (Bragg's law)

Measured reflectivity will depend on angle and wavelength.

Total reflection for angles less than critical angle, $\theta_c = \arccos(n_1/n_2)$

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Useful Physical Ideas

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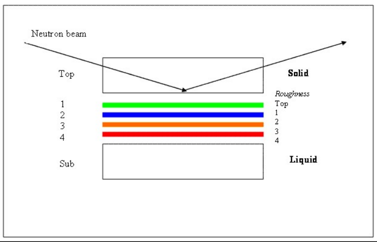
Models for complex interfaces can be constructed from multiple thin layers of different refractive index, n or scattering length density, ρ.



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Useful Physical Ideas

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Isotopes (e.g. D/H substitution) can be used to label particular species or alter contrast

Neutrons have spin – effectively a field dependent contribution to scattering length

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Abeles Optical Matrix Method

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$$r_j = \begin{bmatrix} e^{i\beta_{j-1}} & r_{j-1}e^{i\beta_{j-1}} \\ r_{j-1}e^{-i\beta_{j-1}} & e^{-i\beta_{j-1}} \end{bmatrix}$$

$\beta_j = (2\pi/\lambda) n_j d_j \sin\theta_j$ $\qquad$ $p_j = n_j \sin\theta_j$

$r_j = (p_{j-1} - p_j)/(p_{j-1} + p_j)$ $\qquad$ $M_R = [M_1][M_2]...[M_{n-1}]$

$$R(Q) = M_{21}M_{21}{}^* / M_{11}M_{11}{}^*$$

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Magnetic Contrast

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$b_m = \mu_0 e^2 S \gamma / 4\pi m_e$

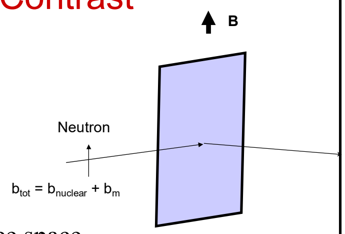
e, electronic charge

m_e, electron mass

S, spin

μ_0, Permeability of free space

γ, gyromagnetic ratio, 1.913



$b_{tot} = b_{nuclear} \pm b_m$

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Magnetic Contrast

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$b_m = \mu_0 e^2 S \gamma / 4\pi m_e$

e, electronic charge

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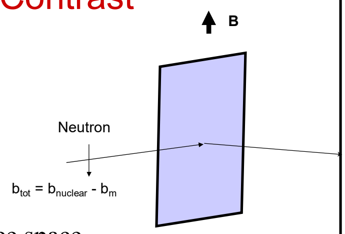

$b_{tot} = b_{nuclear} \pm b_m$

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Scattering and Reflection

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$\rho(Q)$ is Fourier transform of the scattering length density distribution normal to the interface, $\rho(z)$

$$R(Q) = \frac{16\pi^2}{Q^2}\left|\rho(Q)\right|^2$$

$$\rho(Q) = \int_{-\infty}^{\infty} \rho(z) e^{-iQz} dz$$

For sharp interface:

$R(Q) \sim 1/Q^4$

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